

Cosmological constraints from Radial Baryon Acoustic Oscillation measurements and Observational Hubble data

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We use the Radial Baryon Acoustic Oscillation (RBAO) measurements, distant type Ia supernovae (SNe Ia), the observational $H(z)$ data (OHD) and the Cosmic Microwave Background (CMB) shift parameter data to constrain cosmological parameters of Λ CDM and XCDM cosmologies and further examine the role of OHD and SNe Ia data in cosmological constraints. We marginalize the likelihood function over h by integrating the probability density $P \propto e^{-\chi^2/2}$ to obtain the best fitting results and the confidence regions in the $\Omega_m - \Omega_\Lambda$ plane. With the combination analysis for both of the Λ CDM and XCDM models, we find that the confidence regions of 68.3%, 95.4% and 99.7% levels using OHD+RBAO+CMB data are in good agreement with that of SNe Ia+RBAO+CMB data which is consistent with the result of Lin et al's work. With more data of OHD, we can probably constrain the cosmological parameters using OHD data instead of SNe Ia data in the future.

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I. INTRODUCTION

In modern cosmology, the discovery of the accelerating expansion of the universe is a great encouraging development. This result was firstly shown by the observations of the distant SNe Ia [1, 2], which can be seen as a standard candle [3, 4]. Afterwards, the CMB measurement by Wilkinson Microwave Anisotropy Probe (WMAP) [5] and the large scale structure survey by Sloan Digital Sky Survey (SDSS) [6, 7] support the same result as the SNe Ia presented. To explain the acceleration of the universe, many cosmological models were introduced, including the Quintessence [8], the brane world [9], the Chaplygin Gas [10] and the holographic dark energy models [11] and so on. The most popular model is referred as Λ CDM cosmology composed of cold pressureless dark matter with the equation of state (EOS) $w = p/\rho$ and Einstein's cosmological constant Λ which is the most economic and the oldest form of dark energy with $w = -1$ [12]. This model provides a reasonably good fit to most current cosmological data [13, 14]. Additionally, one also consider another cosmological model—XCDM parametrization which is useful in describing the time-varying dark energy models. In this model, the dark energy is assumed to be a perfect fluid with the equation of state (EOS) $\omega = p_x/\rho_x$, where ω is a number less than $-1/3$ [15]. In addition, the $f(R)$ gravity models are also constrained using the statistical lens sample from Sloan Digital Sky Survey Quasar Lens Search Data Release 3 (SQLS DR3) [16].

With the perfect observational data, one can compare the observational results with theoretical predictions of different models and determine which model is better [15, 17]. Besides, another important task of cosmology is to constrain the cosmological parameters of various cosmological models using the redshift-dependent quantities, for example the luminosity distance to a particular class of objects such as SNe Ia and Gamma-Ray Bursts (GRBs) [18]. X-ray gas mass fraction of galaxy clusters is also very popular [19]. Recently, the size of the BAO peak detected in the large-scale correlation function of luminous red galaxies from SDSS [20] and the CMB shift parameter \mathcal{R} obtained from acoustic oscillations in the CMB temperature anisotropy power spectrum [21, 22] are also widely used to constrain cosmological models.

Recently, one method based on the observational Hubble parameter $H(z)$ data as a function of redshift z , which are related to the differential ages of the oldest galaxies has been used to test cosmological models [23–33]. Furthermore, the new observations have given more OHD data [34, 35]. These new released data may improve the constraints of cosmological parameters evidently. Except that, the latest measurements of the radial baryon acoustic oscillation (RBAO) were discussed deeply [15, 36]. Lin et al's work [24] has shown that the constraints using different data can provide some different results but more consistent with each other. And the combinations of varieties of data can also make the constraints tighter [37–39]. Following this direction, we intend to further examine the role OHD played in constraining the cosmological parameters by using RBAO. We compare the constraints on the Λ CDM cosmology and

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Λ CDM cosmology using OHD and SNe Ia data combined with RBAO and CMB data. We find that the OHD plays the same role as SNe Ia for joint cosmological constraints.

Our paper is organized as follows. In Sec.II we describe the observational data we used in this paper. In Sec.III we present the dark energy models. In Sec.IV, we show the constraints. And finally, we give our conclusion.

II. OBSERVATIONAL DATA

A. SNe Ia data

The luminosity distance of Type Ia supernova(SNe Ia), d_L , can be estimated by the relation

$$m = M + 5 \log d_L + 25, \quad (1)$$

where m is the K-corrected observed apparent magnitude and M the absolute magnitude of SNe Ia. The luminosity distance depends on the content and geometry of the Universe in a Friedmann-Robertson-Walker (FRW) cosmology

$$d_L = \frac{c(1+z)}{H_0 \sqrt{|\Omega_k|}} \text{sinn}[\sqrt{|\Omega_k|} \mathcal{F}(z)], \quad (2)$$

where $\text{sinn}(x)$ is $\sinh(x)$ for $\Omega_k > 0$, x for $\Omega_k = 0$, and $\sin(x)$ for $\Omega_k < 0$ respectively. The function $\mathcal{F}(z)$ is defined as $\mathcal{F}(z) = \int_0^z dz/E(z)$ with $E(z) = H(z)/H_0$. $E(z)$ is the expansion rate that has different forms in different cosmological models. H_0 is the Hubble constant and $\Omega_m, \Omega_\Lambda, \Omega_k$ are the matter, cosmological constant and curvature density parameters respectively. The distance modulus is

$$\mu_z = 5 \log \frac{d_L}{10 \text{pc}} = 42.39 + 5 \log \frac{1+z}{h \sqrt{|\Omega_k|}} \text{sinn}[\sqrt{|\Omega_k|} \mathcal{F}(z)], \quad (3)$$

where $h = H_0/100 \text{kms}^{-1} \text{Mpc}^{-1}$. We use the observational SNe Ia data [40] with redshift spans from about 0.01 to 1.75.

B. The Observational $H(z)$ data

In addition, the measurement of the Hubble parameter $H(z)$ is increasingly becoming important in cosmological constraints, and it can be derived from the derivation of redshift z with respect to the cosmic time t [41]

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}, \quad (4)$$

which provides a direct measurement for $H(z)$ through a determination of dz/dt . Jimenez et al demonstrated the feasibility of the method by applying it to a $z \sim 0$ sample[42]. With the availability of new galaxy surveys, it becomes possible to determine $H(z)$ at $z > 0$. By using the different ages of passively evolving galaxies determined from the Gemini Deep Deep Survey (GDDS) [43] and archival data [44–47], Simon et al. derived a set of observational $H(z)$ data [25, 42, 48]. The detailed estimation method can be found in the work [48]. As z has a relatively wide range, $0.1 < z < 1.8$, these data are expected to provide a more full-scale description of the dynamical evolution of our universe. The application of OHD to cosmology can be referred to [23–26, 48].

C. The CMB data

The CMB shift parameter \mathcal{R} is arguably one of the most model-independent parameters among those which can be inferred from CMB data, provided that the dark energy density parameter is negligible at recombination, and it does not depend on H_0 [49, 50]. It is directly proportional to the ratio of the angular diameter distance to the decoupling epoch divided by the Hubble horizon size at the decoupling epoch. That is,

$$\mathcal{R} = \frac{\sqrt{\Omega_m}}{\sqrt{|\Omega_k|}} \text{sinn}[\sqrt{|\Omega_k|} \mathcal{F}(z_s)], \quad (5)$$

where $z_s = 1089$ is the redshift of recombination. The value of \mathcal{R} obtained from acoustic oscillations in the CMB temperature anisotropy power spectrum is $\mathcal{R} = 1.715 \pm 0.021$ [21, 22].

D. The RBAO data

The measurement of the large-scale structure baryon acoustic oscillation (BAO) peak length scale has been found efficient to constrain cosmological parameters [51–54]. Gaztañaga recently used SDSS data to measure the radial baryon acoustic scale in two redshift ranges $z \sim 0.15 - 0.30$. The radial baryon acoustic scale is independent from the previous BAO measurement which was either averaged over all direction or just in the transverse direction. The data used was listed in their Table.I in [36]. Theoretically the radial BAO peak scale is given by

$$\Delta z = H(z)r_s/c, \quad (6)$$

where $H(z)$ is the Hubble parameter at redshift z , r_s is the sound horizon at the time of recombination, and c is the speed of light respectively. r_s can be computed as [15]

$$r_s = \frac{\pi(1+z)d_A(z_s)}{l_s}, \quad (7)$$

where z_s is the redshift of the last-scattering surface. We adopt the WMAP 5-year recommended values $l_s = 302.14 \pm 0.87$, $z_s = 1090.5 \pm 1.0$. While d_A is the angular diameter distance, it can be easily computed in a given cosmological model.

III. COSMOLOGY MODELS

We apply the data listed in Sec.II with the predictions of two cosmological models including dark energy. The models we consider are standard Λ CDM and XCDM parametrization of the dark energy's equation of state. As mentioned above, the difference of the two models is existed in the expansion rate

$$\begin{aligned} E(z) &= \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2} \quad (\Lambda\text{CDM}) \\ E(z) &= \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^{3(1+\omega)}} \quad (\text{XCDM}) \end{aligned} \quad (8)$$

In both of the two models the background evolution is described by two parameters. One is the nonrelativistic matter fractional energy density parameter Ω_m and the other one is a parameter that characterizes the dark energy. For the Λ CDM model, the parameter is the cosmological constant fractional energy density parameter Ω_Λ . In the XCDM parametrization, it is the equation of state parameter ω . In this paper, we assume that the XCDM model is the spatially-flat while in the Λ CDM case, the spatial curvature is allowed to vary, with the space curvature fractional energy density parameter $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$.

IV. COSMOLOGICAL CONSTRAINTS

The purpose of this paper is to examine the role of OHD and SNe Ia data in the constraints on cosmological parameters with different cosmological models. The likelihood for the cosmological parameters can be determined from a $\chi^2(h, \Omega_m, \Omega_\Lambda)$ statistics. For RBAO in Λ CDM as an example,

$$\chi^2(\Omega_m, \Omega_\Lambda, h) = \sum_{i=1,2} (\Delta z_{th,i}(\Omega_m, \Omega_\Lambda, h) - \Delta z_{obs,i})^2 / \sigma_{\Delta z,i}^2, \quad (9)$$

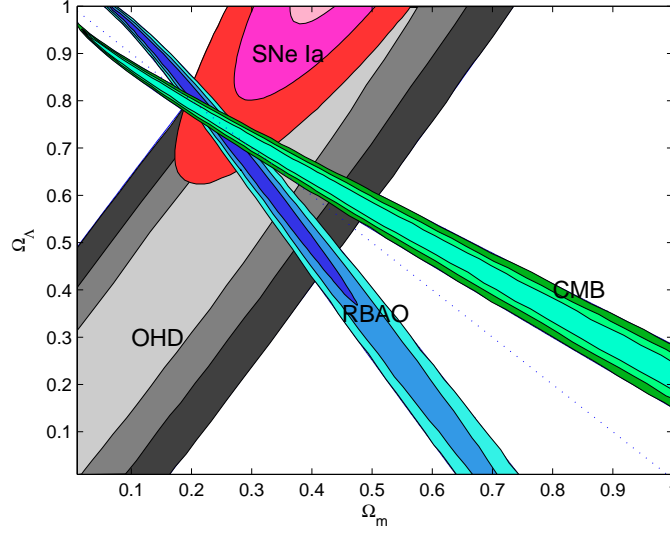


FIG. 1: Confidence regions in the $\Omega_m - \Omega_\Lambda$ plane for the data used alone for Λ CDM model. For each kind of data, with a prior of h , the confidence regions at 68.3%, 95.4% and 99.7% levels from inner to outer are presented respectively. The dotted line demarcates spatially-flat Λ CDM models.

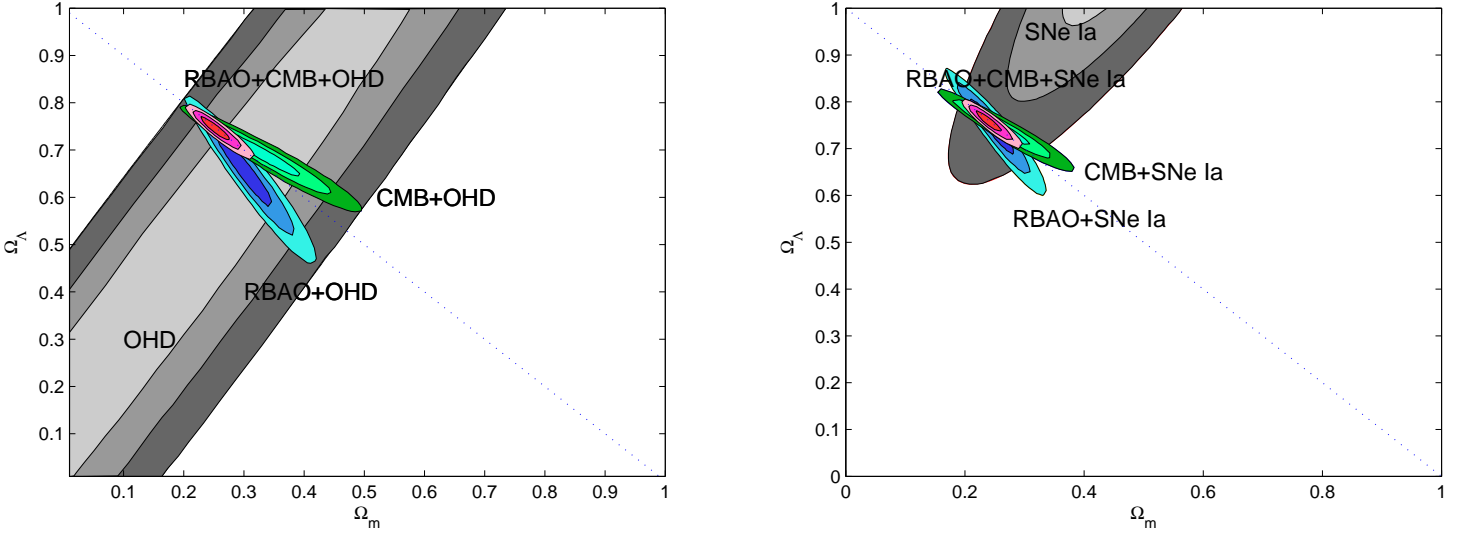


FIG. 2: The confidence regions of the combined constraints of Λ CDM model. *Left panel:* The OHD data combined with RBAO and CMB respectively. And the smallest one correspond to the constraint of the data of RBAO+CMB+OHD. *Right panel:* The SNe Ia data combined with RBAO and CMB respectively. The smallest one indicates the constraint of the data of RBAO+CMB+SNe Ia. The dotted line demarcates spatially-flat Λ CDM models.

to get the constraint results of two parameters Ω_m and Ω_Λ , we marginalize the likelihood functions over h by integrating the probability density $P \propto e^{\chi^2/2}$. Thus we can obtain the best fitting results and the confidence regions in the $\Omega_m - \Omega_\Lambda$ plane. In the calculation, we assume a prior of $h = 0.705 \pm 0.013$ as WMAP5 suggested, and this method can improve the constraints greatly [24]. Fig.1 shows the constraints of each data without any combination for Λ CDM model. It can be easily seen that the confidence regions for each data are almost different. And the best fitting results they give are also different. Both RBAO and CMB data prefer a nearly flat universe.

In order to compare the contribution of OHD and SNe Ia data in constraining the cosmological parameters clearly, it is effective to combine the different data together. Fig.2 presents the combined constraints of OHD and SNe Ia data with RBAO and CMB respectively. It is shown from the Table.I that there are slight differences on the best

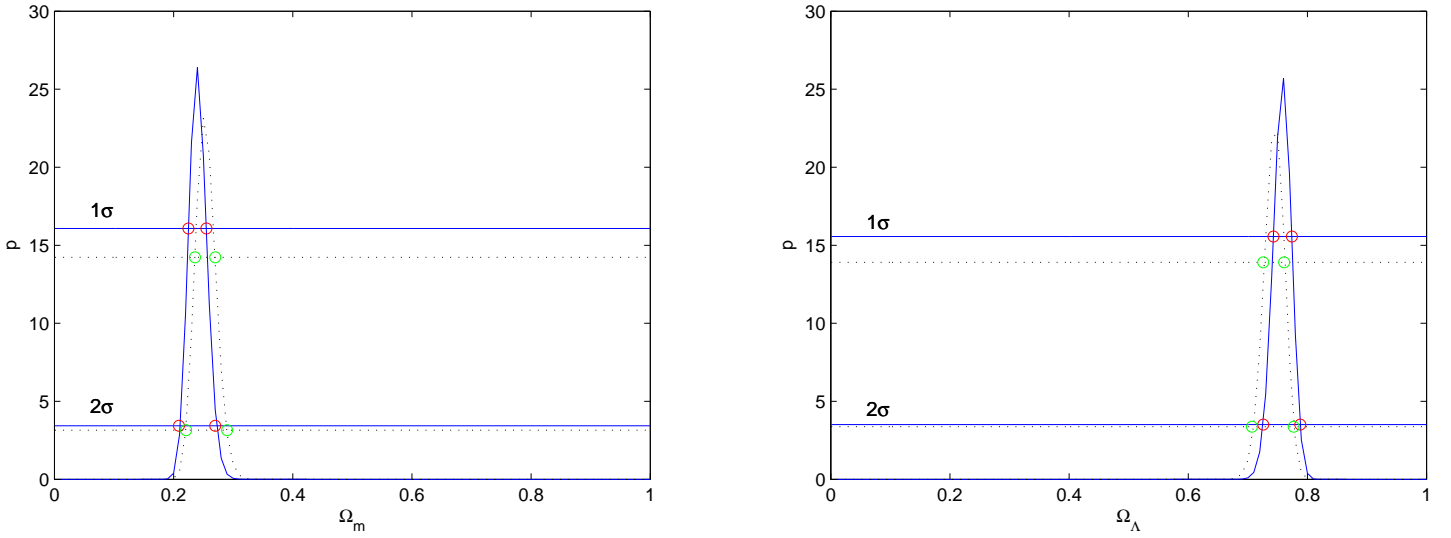


FIG. 3: The one-dimensional probability distribution function(PDF) p with the data of RBAO+CMB+OHD (dotted line) and RBAO+CMB+SNe Ia (solid line) for selections of parameters Ω_m and Ω_Λ with a prior of h for Λ CDM model. The 1 and 2 σ confidence levels are also labeled.

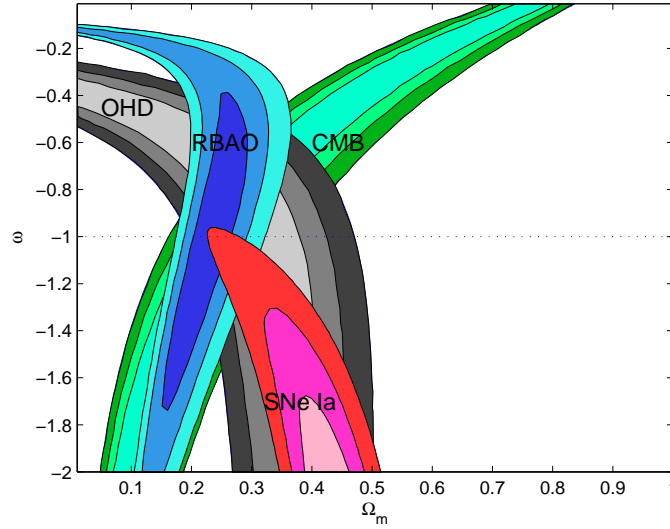


FIG. 4: Same as Fig.1, but for XCDM.

fitting values of Ω_m and Ω_Λ . However, the consistency of the results which is more important indicates that the OHD and SNe Ia data give almost the same contribution in constraining cosmological parameters. We also calculate the one-dimensional probability distribution function(PDF) p for selections of parameters Ω_m and Ω_Λ with a prior of h . Fig.3 presents the PDF of Ω_m and Ω_Λ for RBAO+CMB+OHD and RBAO+CMB+SNe Ia respectively. The 1 and 2 σ confidence levels are also shown. It is easy to see that the most probable value of the two results are roughly consistent with each other.

From the constraints of combined data and the one-dimensional probability distribution function, we can see that some slight discrepancy are shown between the constraints of OHD and SNe Ia combined with other data. However, both the constraints are almost the same restrictive. And their results prefer a nearly flat universe. Applying the data we used above to the XCDM model, first we get Fig.4 that shows the constraints of the alone data used in Fig.1. While Fig.5 shows the combined constraints as Fig.2. It is clear that the constraints of RBAO+CMB+OHD and RBAO+CMB+SNe Ia are both restrictive at the confidence level of 68.3%. The best-fit results of these constraints are

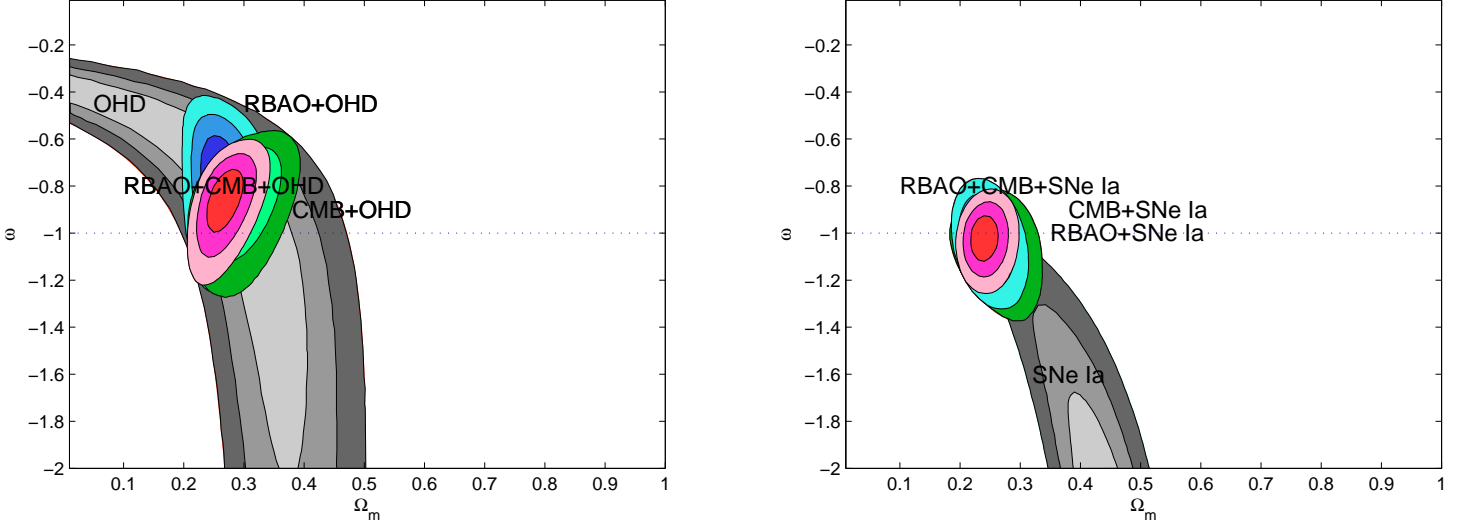


FIG. 5: Same as Fig.2, but for XCDM.

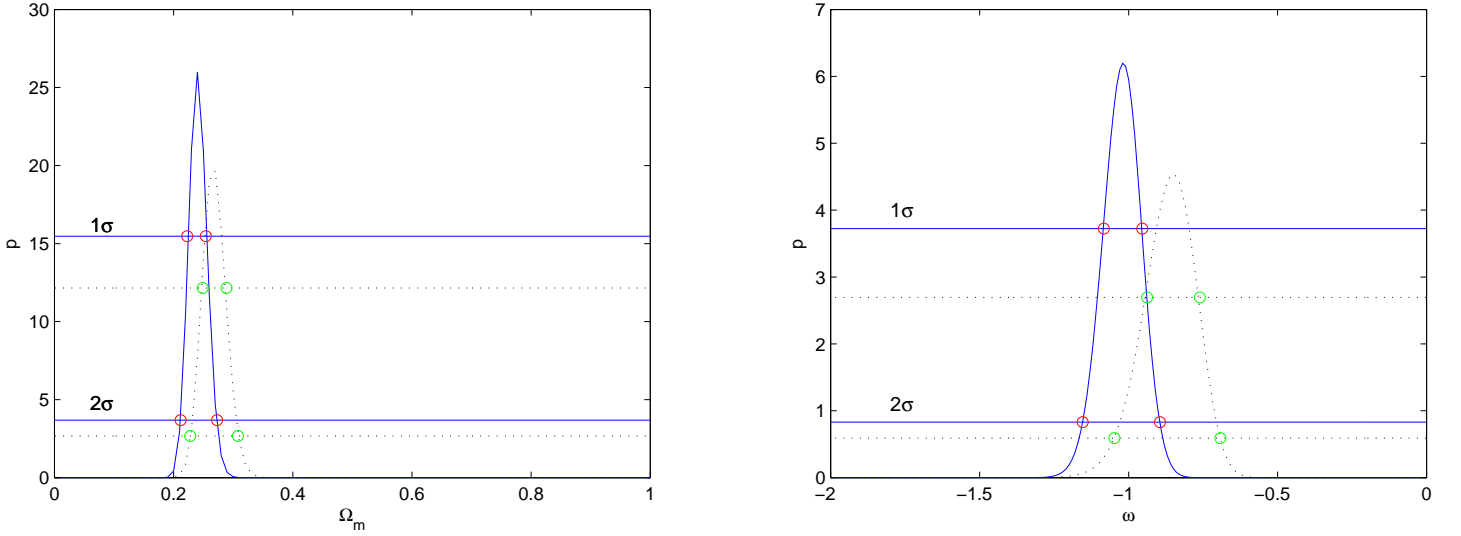


FIG. 6: Same as Fig.3, but for XCDM.

listed in Table.II. In order to examine if the OHD and SNe Ia data play the same role in constraining the cosmological parameters as in Λ CDM model, the one-dimensional probability distribution function(PDF) is also calculated. The PDF curves are plotted in Fig.6. From the results listed above, we can see that the constraints of the parameter Ω_m using the two data combinations are consistent with each other. The main discrepancy is in constraining ω . The 1σ confidence region of ω achieved from RBAO+CMB+OHD is $\omega = -0.84 \pm 0.14$, while RBAO+CMB+SNe Ia suggests $\omega = -1.02 \pm 0.10$. The reason that causes the difference is that the amount of the OHD is so few. With more data achieved in the future, many deficiencies will be improved.

V. DISCUSSIONS AND CONCLUSIONS

Recent observations have provided a lot of information to analyze the dynamical behavior of the universe. Most of them are based on distance measurements, such as SNe Ia. And the present BAO peak scale data and CMB data are

parameters	Ω_m	Ω_Λ
RBAO+OHD	0.30 ± 0.04	0.66 ± 0.07
CMB+OHD	0.32 ± 0.06	0.70 ± 0.04
RBAO+CMB+OHD	0.25 ± 0.02	0.75 ± 0.03
RBAO+SNe Ia	0.25 ± 0.03	0.74 ± 0.06
CMB+SNe Ia	0.25 ± 0.04	0.75 ± 0.03
RBAO+CMB+SNe Ia	0.24 ± 0.02	0.76 ± 0.02

TABLE I: The best-fit results of the Λ CDM model with a prior of h .

parameters	Ω_m	ω
RBAO+OHD	0.25 ± 0.03	-0.75 ± 0.18
CMB+OHD	0.29 ± 0.04	-0.84 ± 0.15
RBAO+CMB+OHD	0.27 ± 0.03	-0.84 ± 0.14
RBAO+SNe Ia	0.24 ± 0.03	-1.02 ± 0.11
CMB+SNe Ia	0.25 ± 0.03	-1.03 ± 0.12
RBAO+CMB+SNe Ia	0.24 ± 0.02	-1.02 ± 0.10

TABLE II: The best-fit results of the XCDM model with the a prior of h .

both sparse and can not provide a tight constraint on dark energy parameters. It is important to use other different probes to set bonds on the cosmological parameters. Following this direction, we used the observational $H(z)$ data from the differential ages of the passively evolving galaxies to constrain the Λ CDM cosmology and XCDM cosmology, combining RBAO and CMB. In order to verify the OHD data can give almost the same contribution in constraining the cosmological parameters as other widely used data, we compared the SNe Ia data in the same way of calculation.

For the Λ CDM universe with a prior of h , the best-fit result of RBAO+CMB+OHD and RBAO+CMB+SNe Ia indicate a nearly flat universe. The constraints of these two data combinations are both very tight and consistent with each other. For the flat XCDM universe with the same prior, there exists some discrepancy in the constraints, especially for the parameter ω , however, the constraint results of Ω_m obtained from the two data combination RBAO+CMB+OHD and RBAO+CMB+SNe Ia are almost the same.

From the above comparison and previous works [23, 25], we find that our results from the observational $H(z)$ data are believable and the computation results are consistent with the results using the data of SNe Ia. So the observational $H(z)$ data can be seen as a complementarity to other cosmological probes. With a large amount of OHD in a wider range of redshift z in the future, we probably can constrain the cosmological parameters using OHD instead or combined with other data.

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